

A SYSTEMATIC STUDY OF FINITE BRST-BFV TRANSFORMATIONS IN GENERALIZED HAMILTONIAN FORMALISM

Igor A. Batalin^{(a)1}, Peter M. Lavrov^{(b,c)2}, Igor V. Tyutin^{(a)3}

^(a) *P.N. Lebedev Physical Institute,
Leninsky Prospect 53, 119 991 Moscow, Russia*

^(b) *Tomsk State Pedagogical University,
Kievskaya St. 60, 634061 Tomsk, Russia*

^(c) *National Research Tomsk State University,
Lenin Av. 36, 634050 Tomsk, Russia*

Abstract

We study systematically finite BRST-BFV transformations in the generalized Hamiltonian formalism. We present explicitly their Jacobians and the form of a solution to the compensation equation determining the functional field dependence of finite Fermionic parameters, necessary to generate arbitrary finite change of gauge-fixing functions in the path integral.

Keywords: generalized Hamiltonian formalism, field-dependent BRST-BFV transformation

¹E-mail: batalin@lpi.ru

²E-mail: lavrov@tspu.edu.ru

³E-mail: tyutin@lpi.ru

1 INTRODUCTION

It is well-known that BRST-BFV symmetry [1, 2, 3, 4, 5] is a powerful tool to study general properties of gauge field systems [6, 7, 8, 9, 10, 11, 12]. Parameters of that symmetry are constant Fermions, although they are allowed to be functionals of fields. Usually, the symmetry is introduced infinitesimally, which means that its Fermionic parameters are considered formally as infinitely-small quantities. Usual strategy is to show that the Jacobian of BRST-BFV transformation does generate arbitrary variation of gauge-fixing functions in the path integral. This can be done by choosing necessary functional dependence of BRST-BFV parameters on fields.

For Yang-Mills theories in Lagrangian formalism based on the Faddeev-Popov method [13] a study of finite field-dependent BRST transformations was initiated in Ref. [14] where a differential equation for the Jacobian of such change of variables in vacuum functional has been proposed. But a solution to this equation has not been found in Ref. [14] and numerous further studies of this problem. Recently [15], it was proved that the problem of finding an explicit form of the Jacobian is pure algebraic and can be solved in terms of the BRST variation of field-dependent parameter. Any finite field dependent BRST transformation of variables in the generating functional of Green functions is related to modification of gauge fixing functional [15, 16].

By following the ideas of paper [14], recently in Ref. [17] an attempt was made to consider finite field-dependent BRST transformations in Hamiltonian formalism within BFV quantization method of dynamical systems with constraints [4, 5, 6]. The same as in Ref. [14], the main result was formulated here as a differential equation for the Jacobian of these transformations. Again, an explicit solution to the differential equation was not found in this approach.

In the present article, we develop systematically the concept of finite BRST-BFV transformations, which means actually that we consider formally BRST-BFV parameters as finite Fermionic quantities. Historically, there were several authors (see [6, 7, 8, 9, 10, 11, 12] and references herein) who worked sporadically with finite BRST-BFV transformations. But the final results were formulated infinitesimally even in these special cases. In the present work, we give a unique consistent approach. Thereby, our new strategy is to show that the Jacobian of these finite transformations does generate an arbitrary finite change of gauge-fixing functions in the path integral. In order to do this, we formulate the compensation equation determining the necessary functional field dependence for finite Fermionic parameters. Then, we present the explicit solution to that compensation equation. We find functional formulation of BRST-BFV transformations and derive the Ward identities as well as a relation connecting generating functionals of Green functions written in two different gauges.

2 FINITE BRST-BFV TRANSFORMATIONS AND THEIR JACOBIANS

Let

$$z^i = (q; p), \quad \varepsilon(z^i) = \varepsilon_i, \quad (2.1)$$

be a complete set of canonical variables specific to the extended phase space of generalized Hamiltonian formalism. The partition function reads

$$Z_\psi = \int Dz \exp \left[\left(\frac{i}{\hbar} \right) W_\psi \right], \quad (2.2)$$

where the action W_ψ is defined as

$$W_\psi = \int \left[\left(\frac{1}{2} \right) z^i(t) \omega_{ik} \dot{z}^k(t) - H(t) \right] dt, \quad H(t) = \mathcal{H}(t) + \{\Omega, \psi\}_t, \quad (2.3)$$

$$\{\Omega, \Omega\} = 0, \quad \{\Omega, \mathcal{H}\} = 0, \quad (2.4)$$

$$\mathcal{H} = H_0 + \dots, \quad \Omega = cT + \dots, \quad (2.5)$$

Here in (2.3), $z^i(t)$ are functions of time (trajectories), $\dot{z}^k(t) = dz^k(t)/dt$, $H(t)$, $\mathcal{H}(t)$, $\Omega(t)$, $\psi(t)$ are local functions of time: $H(t) = H(z)|_{z \rightarrow z(t)}$ and so on, $\{\cdot, \cdot\}_t$ means the Poisson superbracket for fixed time t : $\{\Omega, \psi\}_t = \{\Omega(z), \psi(z)\}|_{z \rightarrow z(t)}$ and so on, $\{z^i, z^k\} = \omega^{ik} = \text{const} = -\omega^{ki}(-1)^{\varepsilon_i \varepsilon_k}$ is an invertible even supermatrix; ω_{ik} ($\omega_{ik} = (-1)^{(\varepsilon_i+1)(\varepsilon_k+1)} \omega_{ki}$) stands for an inverse to ω^{ik} , \mathcal{H} is a Boson with ghost number zero, while Ω and ψ is a Fermion with ghost number $+1$ and -1 , respectively, they are called BRST-BFV generator and gauge Fermion.

It follows from (2.4) that

$$\{\Omega, H\} = 0. \quad (2.6)$$

We define finite BRST-BFV transformations of phase (canonical) variables $z^k \rightarrow \bar{z}^k$ in the form

$$\bar{z}^k = z^k + \{z^k, \Omega\} \mu, \quad (2.7)$$

and then finite BRST-BFV transformations of the trajectories $z^k(t) \rightarrow \bar{z}^k(t)$,

$$\bar{z}^k(t) = \bar{z}^k|_{z \rightarrow z(t)} = z^k(t) + \{z^k, \Omega\}_t \mu, \quad (2.8)$$

where μ is a finite Fermionic parameter. In general, $\mu = \mu[z]$ is a functional of the whole trajectory $z^k(t)$, $-\infty < t < +\infty$. However, μ itself is independent of current time t and phase variables z^k ,

$$d_t \mu = 0, \quad \partial_k \mu = 0, \quad (2.9)$$

where we have denoted

$$d_t = d/dt, \quad \partial_k = \partial/\partial z^k. \quad (2.10)$$

Thus, only a functional derivative $\delta/\delta z(t)$ is capable to differentiate $\mu[z]$ nontrivially.

Now, let us consider the functional Jacobian,

$$\begin{aligned} J &= \text{sDet} \left\{ \bar{z}^i(t) \frac{\overleftarrow{\delta}}{\delta z^j(t')} \right\} = \exp \left\{ \text{sTr} \ln \left[\bar{z}^i(t) \frac{\overleftarrow{\delta}}{\delta z^j(t')} \right] \right\} = \\ &= \exp \left\{ \text{sTr} \ln \left[\delta_j^i \delta(t-t') + \{z^i, \Omega\}_t \left(\mu \frac{\overleftarrow{\delta}}{\delta z^j(t')} \right) + (\{z^i, \Omega\} \overleftarrow{\partial}_j)_t (-1)^{\varepsilon_j} \mu \delta(t-t') \right] \right\}. \end{aligned} \quad (2.11)$$

We factorize the Jacobian (2.12) in the form

$$J = J_1 J_2, \quad (2.12)$$

where

$$J_1 = \exp \left\{ \text{sTr} \ln (G^{-1})_k^i(t, t'; \lambda = 1) \right\}, \quad (2.13)$$

$$(G^{-1})_k^i(t, t'; \lambda) = \delta_k^i \delta(t-t') + \lambda \{z^i, \Omega\}_t \left(\mu \frac{\overleftarrow{\delta}}{\delta z^k(t')} \right), \quad (2.14)$$

$$J_2 = \exp \left\{ \text{sTr} \ln [\delta_k^i \delta(t-t') + (\{z^i, \Omega\} \overleftarrow{\partial}_j)_t G_k^j(t, t'; \lambda = 1) (-1)^{\varepsilon_k} \mu] \right\}. \quad (2.15)$$

It follows from an equation for matrix $G_k^i(t, t'; \lambda)$,

$$G_k^i(t, t'; \lambda) + \lambda \{z^i, \Omega\}_t \int dt'' \left(\mu \frac{\overleftarrow{\delta}}{\delta z^j(t'')} \right) G_k^j(t'', t'; \lambda) = \delta_j^i \delta(t-t'), \quad (2.16)$$

that

$$G_k^i(t, t'; \lambda) = \delta_k^i \delta(t-t') - \lambda \{z^i, \Omega\}_t A_k(t'), \quad (2.17)$$

$$A_k(t') = \int dt'' \mu \frac{\overleftarrow{\delta}}{\delta z^j(t'')} G_k^j(t'', t'; \lambda) = (1 + \lambda \kappa)^{-1} \left(\mu \frac{\overleftarrow{\delta}}{\delta z^k(t')} \right),$$

$$\kappa = \int dt'' \mu \frac{\overleftarrow{\delta}}{\delta z^i(t'')} \{z^i, \Omega\}_{t''}. \quad (2.18)$$

It is a characteristic feature of the factor (2.13) that the operator therein is nontrivial only for μ depending actually on fields. On the other hand, in the factor (2.15), the corresponding operator has a nontrivial part proportional to undifferentiated μ , so that the μ -power series expansion terminates at the quadratic order, as μ itself is nilpotent due to its Fermionic nature. Let us consider the factors (2.13), (2.15) in more detail.

For the J_1 factor, we have

$$\begin{aligned}\ln J_1 &= \int_0^1 d\lambda \int dt dt' G_j^i(t, t'; \lambda) \{z^j, \Omega\}_{t'} \left(\mu \frac{\overleftarrow{\delta}}{\delta z^i(t)} \right) (-1)^{\varepsilon_i} = \\ &= - \int_0^1 d\lambda \int dt' A_j(t') \{z^j, \Omega\}_{t'} = - \int_0^1 d\lambda (1 + \lambda \kappa)^{-1} \kappa = - \ln(1 + \kappa).\end{aligned}\quad (2.19)$$

Then, for the factor J_2 , we have

$$\begin{aligned}\ln J_2 &= \text{sTr} [(\{z^i, \Omega\} \overleftarrow{\partial}_k)_t G_j^k(t, t'; \lambda = 1) (-1)^{\varepsilon_j}] \mu = \\ &= \text{sTr} \left\{ [(\{z^i, \Omega\} \overleftarrow{\partial}_j)_t \delta(t - t') - (\{z^i, \Omega\} \overleftarrow{\partial}_k \{z^k, \Omega\})_t A_j(t')] (-1)^{\varepsilon_j} \right\} \mu.\end{aligned}\quad (2.20)$$

Here in the square brackets in (2.20), the first term is zero due to the Liouville's theorem,

$$\{z^i, \Omega\} \overleftarrow{\partial}_i = \omega^{ik} \partial_k \Omega \overleftarrow{\partial}_i = \omega^{ik} \partial_k \partial_i \Omega = 0. \quad (2.21)$$

The second term in the square bracket in (2.20) is zero due to the Jacobi identity and the first in (2.4),

$$\{\{z^i, \Omega\}, \Omega\} = \left\{ z^i, \left(\frac{1}{2} \right) \{\Omega, \Omega\} \right\} = 0. \quad (2.22)$$

Thus, we arrive at

$$J = J_1 = \exp \left\{ - \ln(1 + \kappa) \right\} = (1 + \kappa)^{-1} \quad (2.23)$$

for the functional Jacobian (2.12) of the finite BRST-BFV transformation (2.7).

3 COMPENSATION EQUATION AND ITS EXPLICIT SOLUTION

Now, we would like to use the Jacobian (2.23) to generate arbitrary finite change $\delta\psi$ of the gauge Fermion ψ in the action (2.3),

$$\psi \rightarrow \psi_1 = \psi + \delta\psi. \quad (3.1)$$

Let us make the transformation (2.8) of the trajectories in the path integral (2.2) for partition function.

First of all, the action (2.3) in the new variables (2.8) reads

$$\bar{W}_\psi = \int \left[\left(\frac{1}{2} \right) \bar{z}^i(t) \omega_{ik} d\bar{z}^k(t)/dt - \bar{H}(t) \right] dt = W_\psi \quad (3.2)$$

where we have used

$$\int [\bar{z}^i(t) \omega_{ik} d\bar{z}^k(t)/dt] dt = \int [z^i(t) \omega_{ik} dz^k(t)/dt] dt + (z^k \partial_k \Omega - 2\Omega)_t \mu \Big|_{-\infty}^{+\infty}, \quad (3.3)$$

and $\bar{H} = H(\bar{z}) = H(z)$ (with eq. (2.6) taken into account). Now, we have for the partition function in the new variables,

$$\begin{aligned} Z_\psi &= \int D\bar{z} \exp \left[\left(\frac{i}{\hbar} \right) \bar{W}_\psi \right] = \int Dz J \exp \left[\left(\frac{i}{\hbar} \right) W_\psi \right] = \\ &= \int Dz \exp \left\{ \left(\frac{i}{\hbar} \right) \left[W_{\psi_1} - \left(- \int dt \{ \Omega, \delta\psi \}_t + \left(\frac{\hbar}{i} \right) \ln(1 + \kappa) \right) \right] \right\}. \end{aligned} \quad (3.4)$$

Let us require the following condition to hold

$$J = \exp \left[- \left(\frac{i}{\hbar} \right) \int dt \{ \Omega, \delta\psi \}_t \right]. \quad (3.5)$$

It follows then

$$Z_{\psi_1} = Z_\psi, \quad (3.6)$$

for arbitrary finite $\delta\psi$. We call the condition (3.5) a "compensation equation". Due to the formula (2.23), it follows that (3.5) is rewritten as

$$\int dt \mu \frac{\overleftarrow{\delta}}{\delta z^i(t)} \{ z^i, \Omega \}_t = \exp \left[\left(\frac{i}{\hbar} \right) \int dt \{ \Omega, \delta\psi \}_t \right] - 1. \quad (3.7)$$

That is a functional equation to determine $\mu[z]$. Introducing a functional x ,

$$x = \left(\frac{i}{\hbar} \right) \int dt \{ \Omega, \delta\psi \}_t = \left(\frac{i}{\hbar} \right) \delta\Psi \int dt \frac{\overleftarrow{\delta}}{\delta z^i(t)} \{ z^i, \Omega \}_t, \quad \delta\Psi = \int dt \delta\psi(t), \quad (3.8)$$

we can rewrite eq. (3.7) in the form

$$\mu \int dt \frac{\overleftarrow{\delta}}{\delta z^i(t)} \{ z^i, \Omega \}_t = f(x) x = \left(\frac{i}{\hbar} \right) [f(x) \delta\Psi] \int dt \frac{\overleftarrow{\delta}}{\delta z^i(t)} \{ z^i, \Omega \}_t, \quad (3.9)$$

where $f(x) = (\exp(x) - 1) x^{-1}$. There is an obvious explicit solution to that equation

$$\mu[\delta\psi] = \mu[z; \delta\psi] = \left(\frac{i}{\hbar} \right) f(x) \delta\Psi. \quad (3.10)$$

Thus we have confirmed explicitly the compensation equation (3.7) to hold.

In the first order in $\delta\psi$, explicit solution (3.10) takes the usual form

$$\mu[\delta\psi] = \left(\frac{i}{\hbar} \right) \delta\Psi + O((\delta\psi)^2). \quad (3.11)$$

4 FUNCTIONAL BRST-BFV TRANSFORMATIONS FOR TRAJECTORIES

It appears quite natural to make our considerations above more transparent by introducing a concept of functional BRST-BFV transformations. Namely, let us define a functional operator (differential) \overleftarrow{d} of the form

$$\overleftarrow{d} = \int dt \frac{\overleftarrow{\delta}}{\delta z^i(t)} \{z^i, \Omega\}_t \quad \varepsilon(\overleftarrow{d}) = 1. \quad (4.1)$$

It follows from the second in (2.4) that the Fermionic operator (4.1) is nilpotent,

$$\overleftarrow{d}^2 = \left(\frac{1}{2}\right) [\overleftarrow{d}, \overleftarrow{d}] = 0. \quad (4.2)$$

The transformation (2.7) can be rewritten in terms of the operator (4.1) as

$$\bar{z}(t) = z(t)(1 + \overleftarrow{d}\mu). \quad (4.3)$$

Thus, the operator (4.1) is a functional BRST-BFV generator. Now, let us define the transformed action,

$$\bar{W}_\psi = W_\psi(1 + \overleftarrow{d}\mu). \quad (4.4)$$

Then, we get exactly the formula (3.2),

$$\bar{W}_\psi = W_\psi + \left(\frac{1}{2}\right) (z^k \partial_k \Omega - 2\Omega)_t \mu \Big|_{-\infty}^{+\infty} = W_\psi. \quad (4.5)$$

As \overleftarrow{d} is linear and μ is nilpotent, applying the operator $1 + \overleftarrow{d}\mu$ to arbitrary functional $F(z)$, $\bar{F}(z) = F(z)(1 + \overleftarrow{d}\mu)$, yields the result $\bar{F}(z) = F(\bar{z})$.

Functional Jacobian (2.23) is rewritten in terms of the generator (4.1) as

$$J = [1 + (\mu \overleftarrow{d})]^{-1}. \quad (4.6)$$

The x in (3.8) can be represented as

$$x = \left(\frac{i}{\hbar}\right) \delta\Psi \overleftarrow{d}, \quad (4.7)$$

and the compensation equation (3.7) takes the form

$$\mu \overleftarrow{d} = \exp \left[\left(\frac{i}{\hbar}\right) (\delta\Psi \overleftarrow{d}) \right] - 1 = \left(\frac{i}{\hbar}\right) [f(x) \delta\Psi] \overleftarrow{d}. \quad (4.8)$$

Thus, we conclude that all the main objects in our considerations can be expressed naturally in terms of a single quantity that is the functional BRST-BFV generator (4.1).

Note that the introduced transformations (2.8), (4.3) form a group. Indeed, let us rewrite (4.3) in the form

$$\bar{z} = z \overleftarrow{T}(\mu), \quad \overleftarrow{T}(\mu) = 1 + \overleftarrow{d} \mu. \quad (4.9)$$

then the composition law of transformations (4.9) reads

$$\overleftarrow{T}(\mu_1) \overleftarrow{T}(\mu_2) = \overleftarrow{T}(\mu_{12}), \quad \mu_{12} = \mu_1 + J_{\mu_1}^{-1} \mu_2, \quad (4.10)$$

where J_{μ_1} is the Jacobian of the transformation (4.9) with μ_1 standing for μ . Indeed, due to the nilpotency (4.2), we have

$$\overleftarrow{T}(\mu_1) \overleftarrow{T}(\mu_2) = 1 + \overleftarrow{d} \mu_1 + \overleftarrow{d} \mu_2 + \overleftarrow{d} \mu_1 \overleftarrow{d} \mu_2 = 1 + \overleftarrow{d} \mu_1 + \overleftarrow{d} \mu_2 + \overleftarrow{d} (\mu_1 \overleftarrow{d}) \mu_2. \quad (4.11)$$

By substituting here

$$\mu_1 \overleftarrow{d} = J_{\mu_1}^{-1} - 1, \quad (4.12)$$

we arrive at (4.10). Moreover, it follows from (4.10) the relation

$$[\overleftarrow{d} \mu_1, \overleftarrow{d} \mu_2] = \overleftarrow{d} \mu_{[12]}, \quad \mu_{[12]} = \mu_{12} - \mu_{21} = -(\mu_1 \mu_2) \overleftarrow{d}. \quad (4.13)$$

5 WARD IDENTITIES DEPENDENT OF BRST-BFV PARAMETERS/FUNCTIONALS

As we have defined finite BRST- BFV transformations, it appears quite natural to apply them immediately to deduce the corresponding modified version of the Ward identity. We will do that just in terms of functional BRST- BFV generator introduced in Section 4.

As usual for that matter, let us proceed with the external-source dependent generating functional,

$$Z_\psi(\zeta) = \int Dz \exp \left[\left(\frac{i}{\hbar} \right) W_\psi(\zeta) \right], \quad W_\psi(\zeta) = W_\psi + \int dt \zeta_k(t) z^k(t), \quad (5.1)$$

where $\zeta_k(t)$ ($\varepsilon(\zeta_k) = \varepsilon_k$) is an arbitrary external source. Of course, in the presence of non-zero external source, the path integral (5.1) is in general actually dependent of gauge Fermion ψ . However, due to the equivalence theorem [18], this dependence has a special form so that physical quantities do not depend on gauge. In its turn, the Ward identity measures the deviation of the path integral from being gauge-independent.

Let us perform in (5.1) the change $z^i \rightarrow \bar{z}^i$ of integration variables, where \bar{z} is defined by (4.3)) with arbitrary $\mu[z]$. Then, by using the gauge invariance (3.2) as well as the (4.6) for the Jacobian, we get what we call a "modified Ward identity",

$$\left\langle \left[1 + \left(\frac{i}{\hbar} \right) \int dt \zeta_k(t) (z^k(t) \overleftarrow{d}) \mu \right] [1 + (\mu \overleftarrow{d})]^{-1} \right\rangle_{\psi, \zeta} = 1, \quad (5.2)$$

where we have denoted the source dependent mean value

$$\langle(\dots)\rangle_{\psi,\zeta} = [Z_\psi(\zeta)]^{-1} \int Dz(\dots) \exp \left[\left(\frac{i}{\hbar} \right) W_\psi(\zeta) \right], \quad \langle 1 \rangle_{\psi,\zeta} = 1, \quad (5.3)$$

related to the source dependent action in the second in (5.1). By construction, in (5.2) both $\zeta_i(t)$ and $\mu[z]$ are arbitrary. The presence of arbitrary $\mu[z]$ in the integrand in (5.2) reveals the implicit dependence of the generating functional (5.1) on the gauge-fixing Fermion Ψ for nonzero external source ζ_i .

For a constant μ , $\mu = \text{const}$, the latter does drop-out completely, and we get from (5.2)

$$\left\langle \int dt \zeta_k(t) (z^k(t) \overleftarrow{d}) \right\rangle_{\psi,\zeta} = 0, \quad (5.4)$$

which is exactly the standard form of a Ward identity. By identifying the μ in (5.2) with the solution of the compensation equation (4.8), it follows according to our result in Sec. 3,

$$Z_{\psi_1}(\zeta) = Z_\psi(\zeta) \left[1 + \left\langle \left(\frac{i}{\hbar} \right) \int dt \zeta_k(t) (z^k(t) \overleftarrow{d}) \mu[-\delta\psi] \right\rangle_{\psi,\zeta} \right]. \quad (5.5)$$

Formula (5.5) generalizes (3.6) to the presence of the external sources.

Finally, let us notice the following. If we introduce the so-called "antifields" $z_i^*(t)$, whose statistics is opposite to that of $z^i(t)$, by adding the term $\int dt z_k^*(t) (z^k(t) \overleftarrow{d})$ to the second in (5.1), we get the new generating functional $Z_\psi(\zeta, z^*)$.

$$Z_\psi(\zeta, z^*) = \int Dz \exp \left[\left(\frac{i}{\hbar} \right) W_\psi(\zeta, z^*) \right], \quad W_\psi(\zeta, z^*) = W_\psi(\zeta) + \int dt z_k^*(t) (z^k(t) \overleftarrow{d}). \quad (5.6)$$

Now, let us perform in (5.6) the change $z^i \rightarrow \bar{z}^i$ with arbitrary $\mu[z]$, we get

$$\left\langle \left[1 + \left(\frac{i}{\hbar} \right) \int dt \zeta_k(t) (z^k(t) \overleftarrow{d}) \mu \right] [1 + (\mu \overleftarrow{d})]^{-1} \right\rangle_{\psi,\zeta,z^*} = 1, \quad (5.7)$$

where

$$\langle(\dots)\rangle_{\psi,\zeta,z^*} = [Z_\psi(\zeta, z^*)]^{-1} \int Dz(\dots) \exp \left[\left(\frac{i}{\hbar} \right) W_\psi(\zeta, z^*) \right], \quad \langle 1 \rangle_{\psi,\zeta,z^*} = 1. \quad (5.8)$$

Due to the nilpotency of \overleftarrow{d} , the only difference between (5.2) and (5.7) is the one between (5.3) and (5.8). For a $\mu = \text{const}$, we get from (5.7)

$$\left\langle \int dt \zeta_k(t) (z^k(t) \overleftarrow{d}) \right\rangle_{\psi,\zeta,z^*} = 0. \quad (5.9)$$

In terms of (5.6), the latter is rewritten in a variation-derivative form

$$\int dt \zeta_k(t) \frac{\overrightarrow{\delta}}{\delta z_k^*(t)} \ln Z_\psi(\zeta, z^*) = 0. \quad (5.10)$$

Now, let $S(z, z^*)$ be a functional Legendre transform to $(\hbar/i) \ln Z_\psi(\zeta, z^*)$ with respect to the external source ζ_i ,

$$z^k = \left(\frac{\hbar}{i}\right) \ln Z_\psi(\zeta, z^*) \frac{\overleftarrow{\delta}}{\delta \zeta_k} (-1)^{\varepsilon_k} = \left(\frac{\hbar}{i}\right) \frac{\overrightarrow{\delta}}{\delta \zeta_k} \ln Z_\psi(\zeta, z^*), \quad (5.11)$$

$$S(z, z^*) = \left(\frac{\hbar}{i}\right) \ln Z_\psi(\zeta, z^*) - \int dt \zeta_k(t) z^k(t), \quad (5.12)$$

$$S(z, z^*) \frac{\overleftarrow{\delta}}{\delta z^j(t)} = -\zeta_j(t). \quad (5.13)$$

It follows then from (5.5)-(5.13) that the master equation,

$$(S, S) = 0, \quad (5.14)$$

holds for S , where the notation $(,)$ on the left-hand side means the so-called "antibracket" well-known in the BV formalism [7] for covariant quantization of gauge field systems,

$$(f, g) = \int dt f \left(\frac{\overleftarrow{\delta}}{\delta z^i(t)} \frac{\overrightarrow{\delta}}{\delta z_i^*(t)} - \frac{\overleftarrow{\delta}}{\delta z_i^*(t)} \frac{\overrightarrow{\delta}}{\delta z^i(t)} \right) g. \quad (5.15)$$

6 Discussions

We have introduced the conception of finite BRST-BFV transformations in the generalized Hamiltonian formalism for dynamical systems with constraints [4, 5, 6]. It was shown that the Jacobian of finite BRST-BFV transformations, being the main ingredient of the approach, can be calculated explicitly in terms of the corresponding generator acting on finite field-dependent functional parameter of these transformations. We have introduced the compensation equation providing for a connection between the generating functionals of Green functions formulated for a given dynamical system in two different gauges. We have found an explicit solution to the compensation equation. We have proposed the functional approach to BRST-BFV transformations, and then reproduced all the results obtained above, in functional terms. The functional formulation of finite BRST-BFV transformations provided for deriving in a simple way the Ward identity and connection between the generating functional of Green functions written in different gauges.

In the present paper we have explored the standard BRST-BFV symmetry in the generalized Hamiltonian formalism [4, 5, 6]. Some years ago, we have proposed [19, 20] the generalized Hamiltonian formalism (which is also known as the $\text{Sp}(2)$ -canonical quantization) based on the extended BRST symmetry. Instead of one Fermionic parameter in the BRST-BFV transformations, in the latter case one has to deal with two Fermionic parameters. It seems very interesting to extend the results obtained above to the case of extended BRST symmetry.

Acknowledgments

I.A.B. would like to thank Klaus Bering of Masaryk University for interesting discussions. The work of I.A.B. is supported in part by the RFBR grants 14-01-00489 and 14-02-01171. The work of P.M.L. is partially supported by the Ministry of Education and Science of Russian Federation, grant TSPU-122, by the Presidential grant 88.2014.2 for LRSS and by the RFBR grant 12-02-00121. The work of I.V.T. is partially supported by the RFBR grant 14-01-00489.

References

- [1] C. Becchi, A. Rouet and R. Stora, *The abelian Higgs-Kibble, unitarity of the S-operator*, Phys. Lett. B52 (1974) 344;
- [2] C. Becchi, A. Rouet and R. Stora, *Renormalization of Gauge Theories* Ann. Phys. (N. Y.) 98 (1976) 287;
- [3] I. V. Tyutin, *Gauge invariance in field theory and statistical physics in operator formalism*, Lebedev Institute preprint No. 39 (1975) (arXiv:0812.0580[hep-th]).
- [4] E. S. Fradkin and G. A. Vilkovisky, *Quantization Of Relativistic Systems With Constraints*, Phys. Lett. B55 (1975) 224.
- [5] I. A. Batalin and G. A. Vilkovisky, *Relativistic S-matrix of dynamical systems with boson and fermion constraints*, Phys. Lett. B69 (1977) 309.
- [6] E. S. Fradkin and T.E. Fradkina, *Quantization of Relativistic Systems with Boson and Fermion First and Second Class Constraints*, Phys. Lett. B72 (1978) 343.
- [7] I. A. Batalin and G. A. Vilkovisky, *Gauge algebra and quantization*, Phys. Lett. B102 (1981) 27.
- [8] B.L. Voronov, P.M. Lavrov and I.V. Tyutin, *Canonical transformations and gauge dependence in general gauge theories*, Sov. J. Nucl. Phys. 36 (1982) 292.
- [9] P.M. Lavrov and I.V. Tyutin, *Gauge theories of general form*, Sov. Phys. J. 25 (1982) 639.
- [10] I. A. Batalin and E. S. Fradkin, *A Generalized Canonical Formalism and Quantization of Reducible Gauge Theories*, Phys. Lett. B122 (1983) 157.
- [11] M. Henneaux, *Hamiltonian Form of the Path Integral for Theories with a Gauge Freedom*, Phys. Rep. 126 (1985) 1.
- [12] M. Henneaux and C. Teitelboim, *Quantization of gauge systems*, Princeton U.P., Princeton (1992).
- [13] L.D. Faddeev and V.N. Popov, *Feynman diagrams for the Yang-Mills field*, Phys. Lett. B25 (1967) 29.
- [14] S. D. Joglekar and B. P. Mandal, *Finite field dependent BRS transformations*, Phys.Rev. D51 (1995) 1919.

- [15] P. M. Lavrov and O. Lechtenfeld, *Field-dependent BRST transformations in Yang-Mills theory*, Phys. Lett. B725 (2013) 382.
- [16] P. M. Lavrov and O. Lechtenfeld, *Gribov horizon beyond the Landau gauge*, Phys. Lett. B725 (2013) 386.
- [17] S. K. Rai and B. P. Mandal, *Finite Nilpotent BRST transformations in Hamiltonian formulation*, Int.J.Theor.Phys. 52 (2013) 3512.
- [18] R. E. Kallosh and I.V. Tyutin, *The equivalence theorem and gauge invariance in renormalizable theories*, Sov. J. Nucl. Phys. 17 (1973) 98.
- [19] I. A. Batalin, P. M. Lavrov and I. V. Tyutin, *Extended BRST quantization of gauge theories in generalized canonical formalism*, J. Math. Phys. 31 (1990) 6.
- [20] I. A. Batalin, P. M. Lavrov and I. V. Tyutin, *An $Sp(2)$ covariant version of generalized canonical quantization of dynamical system with linearly dependent constraints*, J. Math. Phys. 31 (1990) 2708.